A METHOD FOR DETERMINING THE OPTIMAL ORIENTATION OF FLAT-PLATE SOLAR COLLECTORS

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ABSTRACT

To estimate the optimal orientation of a solar collector located in an urban environment, simulations of the shortwave irradiance on a collector, as modified by the presence of buildings, were performed. These simulations show that typical "rule-of-thumb" collector orientations may result in significantly less than maximum energy receipt.

INTRODUCTION

The amount of solar energy available to a fixed, flat-plate solar collector is highly dependent on the orientation of the collector and the nature of its surroundings as well as on climatic conditions and solar geometry. Asymmetries in available global irradiance--caused, in part, by non-symmetric diurnal or seasonal cloud cover—and variations in the surrounding environment add considerable complexity to the problem. Non-uniform reflectance or uneven horizons, for instance, can create distributions of total irradiance on flat-plate solar collectors that cannot be predicted by considerations of solar geometry alone. This is contrary to the commonly accepted "rule-of-thumb",collector-siting methods which generally face the collector due South (in the northern hemisphere), and tilt it as a function of the latitude of the site and the period of the year for which maximum energy receipt is desired. In environments dominated by asymmetries in global irradiance or by non-uniform terrain characteristics—including the presence of trees and/or buildings— optimal collector orientation can be determined better by simulating the fluxes of direct, diffuse and reflected radiation than by the use of these rule-of-thumb orientations.

A numerical model to simulate solar irradiance on flat-plate solar collectors — or any planar surface—has been developed [1] and can be used to investigate the effects of horizonal obstructions and non-uniform surroundings on total irradiance and optimal orientation of flat-plate collectors. The model is briefly described in the next section of this paper, followed by an example of its use for estimating the shortwave fluxes on collectors of various orientation situated in a hypothetical urban environment erected at Boston.

IRRADIANCE COMPUTATIONS

For the simulations presented here, the availability of hourly values of global irradiance—either observed of modeled—is assumed and these values will be used to estimate the direct beam, isotropic sky-diffuse and anisotropic sky-diffuse portions of global irradiance. The components of global irradiance are subsequently modified to account for collector orientation and the shadowing effects of horizonal obstructions, if any are present. In addition, estimates are made of the irradiance reflected from both the ground and any obstructions onto the collector.

Determination of irradiance components

Various methods of determining the diffuse fraction of global irradiance have been developed, beginning with Liu and Jordan [2]. Their relationship for the daily diffuse fraction is based on the concept of a clearness index defined as that proportion of extraterrestrial radiation that reaches the ground. Several researchers have used the clearness index to develop similar relationships for hourly computations of the diffuse fraction of global irradiance. Erbs [3], for instance, developed a relationship based on over 19,000 observations of hourly diffuse and global irradiance at four locations in the United States. His formulation is given by

$$
H_{hd}/H_h = \begin{cases} 1.0-0.09k_T & k_T \le 0.22 \\ 0.9511-0.1604k_T+4.388k_T^2 & \\ \quad -16.638k_T^3+12.336k_T^4 & 0.22 < k_T \le 0.80 \\ 0.165 & k_T > 0.80 \end{cases} \tag{1}
$$

where H_{hd} is total sky-diffuse irradiance on a horizontal surface, H_h is global irradiance and k_T is the hourly clearness index (H_h/H_o). Because equation (1) is thought to be superior to other such relationships—especially when used in the conterminous United States [4]—it is used in the present study to determine the diffuse fraction of global irradiance and, hence, total sky-diffuse irradiance on a horizontal surface. The remaining portion of global irradiance, therefore, is horizontal direct beam irradiance (i.e., $H_{hb}=H_h-H_{hd}$). In order to account for tilted collector surfaces, each of the irradiance components must be adjusted for non-horizontal orientation. Direct beam irradiance on any surface is given by

$$
H_{ib}=H_{nb}\cos\theta_i \tag{2}
$$

where H_{nb} is normal-incidence direct beam irradiance ($H_{nb}=H_{hb}/\cos\theta_z$, where θ_z is the solar zenith angle) and θ_i is the angle between the surface normal and the sun's rays.

Total, sky-diffuse irradiance is not, in general, uniform over the sky hemisphere [5-10], but is composed of isotropic and anisotropic components which must be estimated prior to the consideration of the diffuse irradiance on sloping surfaces. Hay [11] developed anisotropy indices which can be used to estimate the proportion of sky-diffuse irradiance that is anisotropic. For any surface, the anisotropy index (κ_i) is given by

$$
\kappa_i=(H_{nb}/H_{sc})\cos\theta_i \tag{3}
$$

where H_{sc} is the solar constant. By assuming that all anisotropic radiation is circumsolar, anisotropic sky-diffuse (H_{ida}) and isotropic sky-diffuse (H_{idi}) can be expressed as

$$
H_{ida}=H_{hd}\kappa_i/\cos\theta_z \tag{4}
$$

and

$$
H_{idi}=H_{hd}(1-\kappa_h/\cos\theta_z)[(1+\cos\beta)/2] \tag{5}
$$

where again θ_z is the solar zenith angle, κ_h is the anisotropy index for a horizontal surface and β is the tilt angle of the collector. When considering non-horizontal surfaces, an additional component of total irradiance on the collector surface must be included—reflection from the ground. If the ground is assumed to be a level, perfectly diffuse reflector of constant albedo(ρ), ground-reflected irradiance on the collector is given by

$$
H_{idr}=H_h\rho[(1-\cos\beta)/2]. \tag{6}
$$

Therefore, in the absence of horizonal obstructions and under the assumptions outlined above, total irradiance on a flat-plate solar collector can be estimated by the sum of the irradiance components:

$$
H_i=H_{ib}+H_{idi}+H_{ida}+H_{idr}. \tag{7}
$$

When the collector is located in an obstructed environment, each of these irradiance components must be modified to account for the presence of the obstructions.

Obstruction effects

The most obvious effect of horizonal obstructions on the irradiance of flat-plate solar collectors is the total eclipsing of direct beam irradiance (i.e., shadowing) and this is often the only obstruction effect considered [12-15]. In addition, however, obstructions also will reduce the sky-diffuse and ground-reflected irradiances and they may augment total collector irradiance by reflection from the obstructions onto the collector surface. In order to simplify the computation of obstruction effects somewhat, the present model assumes that 1) the collector has one edge along the ground, 2) the collector is small compared to the obstructions (i.e., can be considered a differential element), 3) the obstructions are rectangular and of constant albedo on each face and 4) the ground is level and of constant albedo. Non-rectangular obstructions or obstructions of non-constant albedo can be included by merely subdividing the obstruction into smaller, more homogeneous rectangles. Because the model assumes that all anisotropic sky-diffuse irradiance is circumsolar, the contributions to total irradiance of both direct beam and anisotropic sky-diffuse irradiance terms are zero when the sun is behind an obstruction. Because isotropic sky-diffuse and ground-reflected diffuse irradiances are not point sources, the reduction in the irradiance from these sources will decrease proportionally to the reduction in the area of sky and ground, respectively, which is seen by the collector when no obstructions are present. For isotropic sky-diffuse irradiance, this reduction can be expressed as a function of the total collector view factor (see [16]) with all obstructions and the collector tilt [1]. Isotropic sky diffuse irradiance on the collector after correction for horizonal obstructions then can be given as

$$H'_{idi}=H_{idi}\left\{1-\left[\sum_{j=1}^{n}F(\Delta A_j\rightarrow dA_i)\right]/\left[(1+\cos\beta)/2\right]\right\} \tag{8}$$

where $F(\Delta A_j\rightarrow dA_i)$ is the view factor between the j-th obstruction and the collector.

Since it is assumed that the collector has one edge on a level ground surface, only those obstructions that also have an edge along the ground need be considered when computing the reduction in ground-reflected irradiance due to the presence of obstructions. These obstructions obscure some portion of an infinite semicircle seen by the collector [1] and the collector, therefore, receives no ground-reflected irradiance from those areas. In order to estimate the ground area behind any obstructions and, hence, the proportion of ground-reflected irradiance which is blocked by the obstructions, it is necessary to place a limit on the size of the semicircle. Since the view factor between a vertical collector and the horizontal ground surface will be the maximum for any collector tilt considered here (i.e., tilts greater than 90° are not considered), we can determine a distance at which the view factor and, therefore, the reflected irradiance contribution, is negligible. For the present study, a radius of 50 m (for which the view factor between a vertical collector and $1m^2$ of ground is less than 10^{-6}) was used to determine obstruction effects on ground-reflected collector irradiance. The modified, ground-reflected component of total irradiance on the collector is expressed as

$$H'_{idr}=H_{idr}\left\{1-\left[\left(\sum_{j=1}^{n}G_j\right)/(\pi r^2/2)\right]\right\} \tag{9}$$

where G_j is the area of that portion of the semicircle of radius r (50 m) that is behind the j-th obstruction. Other effects of obstructions on the ground-reflected irradiance—such as shadowing of those areas on the ground that do reflect onto the collector—are not considered here.

Finally, an additional source of collector irradiance—reflection from the obstructions onto the collector—must be considered. This involves determining the irradiance on each obstruction and the view factor between the obstruction and the collector. Since the obstructions are, in general, large compared to the collector and some obstructions may be partially or completely in the shadow of other obstructions, each obstruction is partitioned into smaller elements and the irradiance on each element and its view factor with the collector are considered. In determining the irradiance on the obstruction elements, equations (2)-(6) are used and each obstruction element is treated as if it were a collector. The direct beam and anisotropic sky-diffuse terms are again set to zero if the obstruction element is shadowed by another obstruction, but no correction of the isotropic sky- and ground-reflected diffuse is made and multiple reflections between obstructions or between obstructions and the ground are ignored. Total irradiance on an obstruction element is estimated by

$$H'_{ijk}=H'_{ibjk}+H'_{idajk}+H_{idijk}+H_{idrjk} \tag{10}$$

where H'_{ibjk} and H'_{idajk} are the corrected direct beam and anisotropic sky-diffuse irradiances, respectively, while H_{idijk} and H_{idrjk} are, respectively, the uncorrected isotropic sky- and ground-reflected diffuse irradiances of the k-th element of the j-th obstruction. Total irradiance on the collector due to reflection from the horizonal obstructions can be found by

$$H_{idro}=\sum_{j=1}^{n}\rho_j\sum_{k=1}^{m}H'_{ijk}F(\Delta A_{jk}\rightarrow dA_i) \tag{11}$$

where ρ_j is the albedo of the j-th obstruction. Total irradiance on a collector, following corrections for the presence of horizonal obstructions, can be expressed as the sum of the corrected irradiance components:

$$H'_i=H'_{ib}+H'_{ida}+H'_{idi}+H'_{idr}+H_{idro}\,. \tag{12}$$

SIMULATION RESULTS

To illustrate the model outlined above, exemplary computations of the irradiance on a flat-plate solar collector at various orientations were performed for both unobstructed and obstructed horizons. Observations of hourly total extraterrestrial and global irradiance as well as a snow cover indicator—the necessary climatic inputs to the model—were obtained from the Typical Meteorological Year (TMY) data set for Boston [17]. In order to investigate the seasonal variation of collector irradiance, three weeks centered on the winter solstice, vernal equinox and summer solstice, respectively, were chosen as representative of a range of solar geometries.

In addition to the climatic inputs, it is necessary to define the collector's physical environment. For an unobstructed collector, this consists of simply specifying the albedo of the ground but, in the obstructed case, the location, size and albedo of each obstruction must also be specified, as well as the view factors between the collector and the obstructions.

For both unobstructed and obstructed scenarios, the ground was assigned an albedo of 0.15, unless the ground was snow-covered, in which case the albedo was set to 0.50—typical of aging or patchy snow. The obstructed calculations were performed using a hypothetical urban environment based on the "arbitrary block model" of Frank, *et al.* [18] with each obstruction assumed to have an albedo of 0.25—representative of various building materials. The view from a horizontal collector indicates the degree to which the buildings obstruct the sky dome and the solar path (Figure 1).

To study the relationships between collector orientation and total irradiance on a collector for both unobstructed and obstructed horizons, hourly irradiance computations were performed at 10° increments for collector azimuths from 120° to 240° and tilts from 0° to 90°. Results then were integrated over each of the three weekly simulation periods. My findings are summarized as isoline maps in tilt-azimuth space of total collector irradiance for each week, expressed as a percentage of total irradiance on an optimally oriented collector (Figure 2). For the unobstructed scenarios, the total irradiance distributions for winter and spring are due to solar geometry as expected. They almost are symmetric about South, with a winter maximum associated with a 60° collector tilt, i.e., approximately 20° greater than the latitude of Boston (42°21′N), while, for spring, the optimal tilt is about equal to the site latitude. In the summer simulation, however, the irradiance distribution has become azimuthally asymmetric and the maximum irradiance is on a collector oriented 35° west of South at a tilt more than 30° less than the site latitude. When simulations with obstructions are performed, the total irradiance distributions generally exhibit asymmetries and the optimal orientation moves away from due South— in addition to the expected decrease in total energy receipt.

Estimates of the total irradiance on a collector oriented by any of the commonly used rules-of-thumb can be obtained—by interpolation—from the output tables of the simulation. Values of total irradiance for three rule-of-thumb orientations, expressed as a proportion of the total irradiance on the optimally-

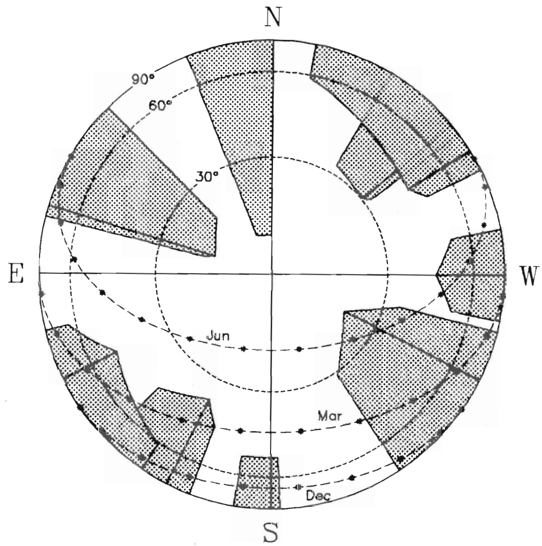

Figure 1: View of the sky dome from a horizontal collector. The path of the sun and its position at the middle of each hour is shown for a representative day of each simulation period.

oriented collector, were computed (Table I) to give an indication of the possible energy losses when the collector is oriented using a simple, rule-of-thumb siting procedure.

Table I: Total irradiance on a collector sited using three rules-of-thumb, expressed as a proportion of total irradiance on an optimally oriented collector as determined by simulation.

Siting Method	Unobstructed			Obstructed		
	December	March	June	December	March	June
latitude-15°	0.848	0.967	0.985	0.943	0.913	0.982
latitude	0.947	0.999	0.870	0.991	0.977	0.858
latitude+15°	0.995	0.974	0.746	0.981	0.968	0.719

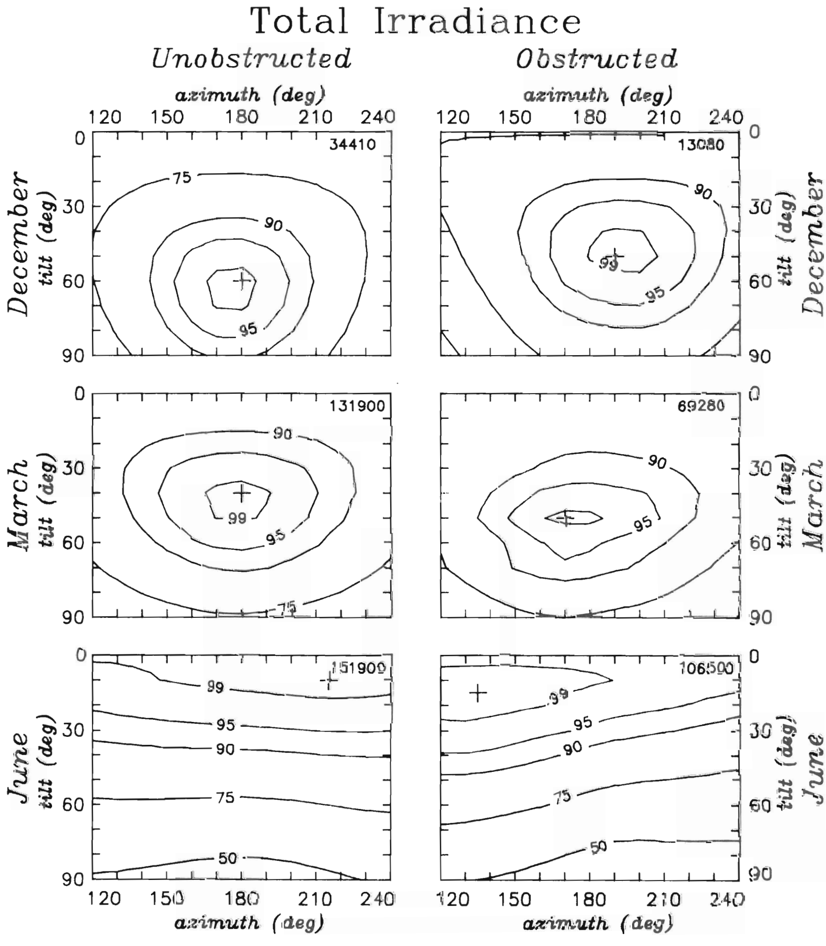

Figure 2: Isoline maps of total collector irradiance integrated over the simulation period, expressed as a percentage of total irradiance on the optimally oriented collector. Total irradiance on the optimally oriented collector is given in the upper right corner of each map (kJm^{-2}) and its position in tilt-azimuth space is marked by a cross.

SUMMARY AND CONCLUSIONS

Simulations of the total irradiance on flat-plate solar collectors at different orientations were performed for both unobstructed and obstructed environments in order to examine the climatic and site dependencies of the optimal orientation of solar collectors. These simulations show that the optimal azimuth may be as much as 45° away from South in an environment with a few, simple obstructions, while the optimal tilt may be 10° different than that obtained by considering solar geometry alone. Energy losses incurred by using rule-of-thumb siting procedures can be significant, relative to an optimally oriented collector determined by simulation—especially when the horizon is obstructed.

Although a few of the model's assumptions and restrictions may be somewhat crude, this effort represents a early step toward the realistic optimization of flat-plate solar collector orientation in an urban environment. Future research and increased computational power will allow the replacement of most of these assumptions with more realistic parameterizations so that future modeling efforts can more accurately evaluate collector irradiance in a variety of situations.

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